Finding Probabilities for $\bar{x}$

1. A bottling company uses a filling machine to fill plastic bottles with a popular cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a normal distribution with mean m = 303 ml and standard deviation s = 3 ml.

    (a) What is the probability that an individual bottle contains less than 300 ml?

    (b) Now take a random sample of 10 bottles. What are the mean and standard deviation of the sample mean contents $\bar{x}$ of these 10 bottles?

    (c) What is the probability that the sample mean contents of the 10 bottles is less than 300 ml?

2. The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 264 days and standard deviation 16 days. Consider 15 pregnant women from a rural area. Assume they are equivalent to a random sample from all women.

    (a) What are the mean and standard deviation of the sample mean length of pregnancy $\bar{x}$ of these 15 pregnancies?

    (b) If we want to predict, with 90% accuracy, the sample mean length of pregnancy for 15 randomly selected women, what values do we use?

    (c) What's the probability the sample mean length of pregnancy lasts less than 250 days? (Contrast this with the probability a single pregnant women is pregnant for less than 250 days, which is 0.1908.)

    (d) Toxic waste is believed to have affected the health of residents of this area. Suppose the sample mean length of pregnancy is indeed 250 days; use the result of part (c) to argue that the waste has an effect of length of pregnancy.

3. The weights of the eggs produced by a certain breed of hen are normally distributed with mean 65 grams and standard deviation of 5 grams.

    (a) What is the probability that one egg selected at random from a hen house will weigh more than 68 grams?

    (b) Consider a carton of 12 eggs to be a simple random sample (SRS) of hen's eggs. If you were to take a large number of repeated samples of size n = 12, what would the mean and standard deviation be of these sample means?

    (c) What is the probability that the average weight of the 12 eggs in a carton selected at random will be more than 68 grams?

    (d) Explain why the numbers from (a) and (c) are so different, using the Central Limit Theorem.

4. In a study done on the life expectancy of 500 people in a certain geographic region, the mean age at death was 72 years and the standard deviation was 5.3 years.

    (a) What is the probability that an individual selected at random will be less than 70 years old?

    (b) If a sample of 50 people from this region is selected, and the probability that the mean life expectancy will be less than 70 years.

    (c) In your own words, explain so that someone not in this class can understand why there is a difference between (a) and (b). Feel free to use specific examples.