Section 11: Probability

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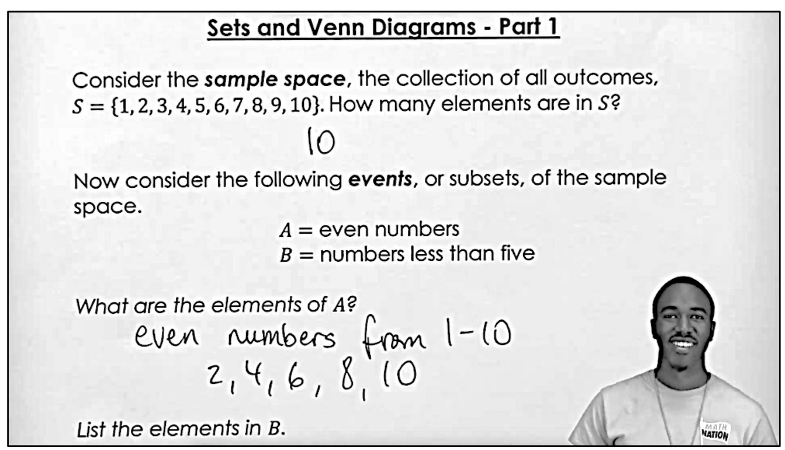


The following Mathematics Florida Standards will be covered in this section:
S-CP.1.1 - Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
S-CP.1.2 - Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
S-CP.1.3 - Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A and the conditional probability of B given A is the same as the probability of B.
S-CP.1.4 - Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
S-CP.1.5 - Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
S-CP.2.6 - Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.
S-CP.2.7 - Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Section 11: Probability
Section 11 – Topic 1
Sets and Venn Diagrams - Part 1

Consider the ***sample space***, the collection of all outcomes, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. How many elements are in S?

Now consider the following ***events***, or subsets, of the sample space.

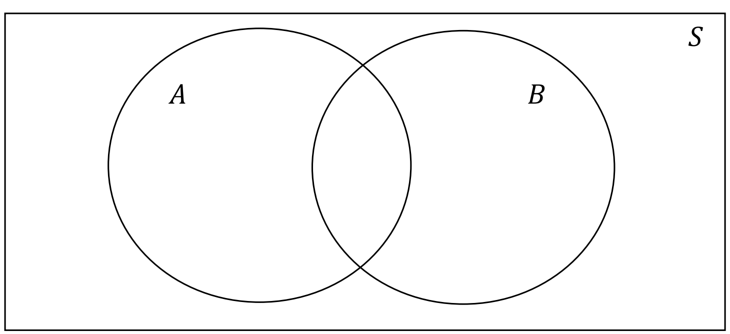
$A =$ even numbers
$B =$ numbers less than five

What are the elements of A?

List the elements in B.

A Venn diagrams can be used to represent various subsets of a sample space, S.

Use the following to represent sets A and B using a Venn diagram. Recall $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.



What elements are in A AND B?

- This is called the ***intersection*** of A and B.
- It is written as $A \cap B$.

What elements are in A OR B?

- This is called the ***union*** of A and B.
- It is written as $A \cup B$.

What elements are NOT in A?

- This is called the ***complement*** of A.
- It is written as $\sim A$.

How many elements are in $\sim(A \cup B)$? Explain what this notation represents.

Define two new events, C and D, so that the sample space S is the union of events C and D.

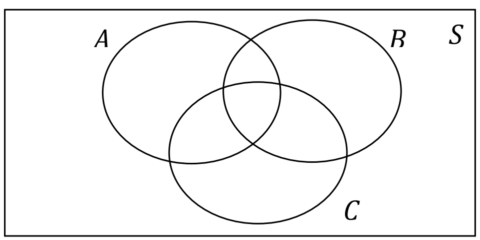
Let's Practice!

1. Consider a standard deck of 52 playing cards. There are 52 cards in a standard deck of cards; half are red (hearts and diamonds), half are black (spades and clubs), and 12 are "face" cards (jacks, queens and kings). Aces are considered to have a value of one.

A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦

Consider the following events:
$A =$ the set of red cards
$B =$ the set of even cards
$C =$ the set of cards less than 5

Part A: Complete the following Venn Diagram with the number of elements in each section.



Part B: How many elements are in the union of events A, B, and C?

Part C: How many cards are in the set $\sim(A \cap C)$?

Section 11 – Topic 2
Sets and Venn Diagrams – Part 2

Try It!

1. The following results were found from a recent survey of 250 subscribers to a conspiracy theory web site:

   - 125 believe we never landed on the moon (Set A).
   - 175 believe 9/11 was a government plot (Set B).
   - 65 believe both theories are true.

   a. Draw a Venn diagram to represent the situation.

   b. How many people are in $A \cap \sim B$?

   c. How many people are in $\sim A \cap B$?

   d. How many people are in $\sim(A \cap B)$?

BEAT THE TEST!

1. Algebra Nation surveyed nine people. The results are summarized below:

Name	Gender	Prefers Cats or Dogs
Amy	Female	Cats
Ashley	Female	Cats
Brian	Male	Dogs
Chelsea	Female	Dogs
Darnell	Male	Dogs
Ethan	Male	Cats
Jose	Male	Dogs
Rachelle	Female	Cats
Stephanie	Female	Dogs

Two people are randomly chosen from those surveyed.

- ➢ Event A: Both people chosen are female
- ➢ Event B: One person who prefers cats and one person who prefers dogs are chosen.

Choose the sets of people that are in the complement of the intersection of events A and B. Select all that apply.

- ☐ Amy and Ashley
- ☐ Amy and Stephanie
- ☐ Brian and Ethan
- ☐ Ashley and Ethan
- ☐ Darnell and Jose
- ☐ Chelsea and Rachelle

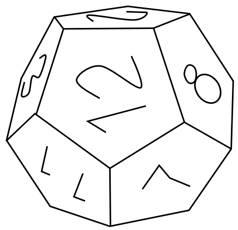
2. Fifty students were surveyed and asked if they are taking a social science (SS), humanities (HM), or a natural science (NS) course the next quarter.

- 21 students are taking a SS course.
- 26 students are taking a HM course.
- 19 students are taking a NS course.
- 9 students are taking SS and HM.
- 7 students are taking SS and NS.
- 10 students are taking HM and NS.
- 3 students are taking all three courses.
- 7 students are not taking any of the courses.

Construct a Venn diagram to represent the situation.

Section 11 – Topic 3
Probability and the Addition Rule – Part 1

Consider the following dodecahedral die, which is a 12-faced die where each face is numbered from 1 to 12.



When the die is rolled, how many total possible results could occur?

How many ways can an even number be rolled?

The ***probability of an event*** could be expressed with the following formula.

- $P(Event) = \frac{\#\ of\ outcomes\ in\ the\ event}{\#\ of\ outcomes\ in\ the\ sample\ space}$

- The probability of an event happening **must be** between 0 and 1 inclusive.

Why do you think a probability can not be greater than 1 or less than 0?

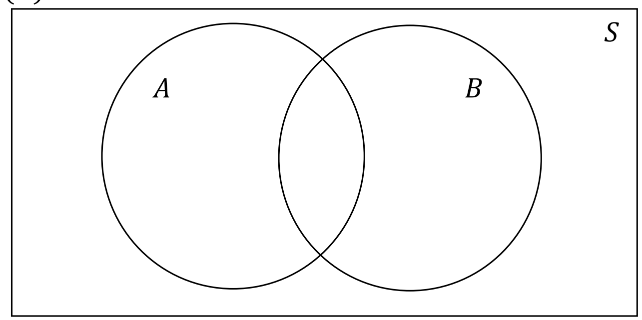
What is the probability of rolling an even number on the first roll?

What is the probability of rolling a number greater than eight?

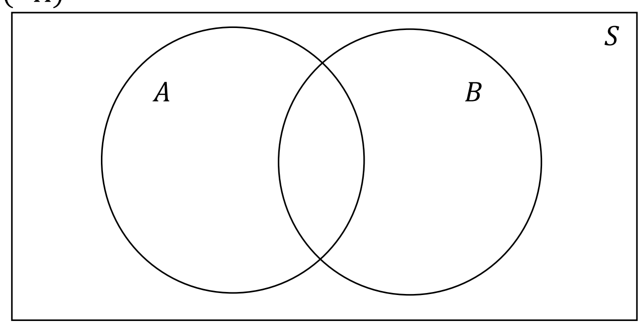
Probabilities can also be found using Venn diagrams.

Suppose event A = {rolling an even number} and event B = rolling a number greater than 8. Use the Venn diagrams below to find each probability and a generic rule for finding the probability. Interpret each probability.

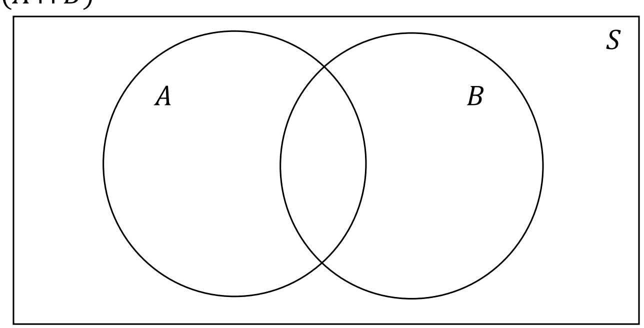
$P(A)$



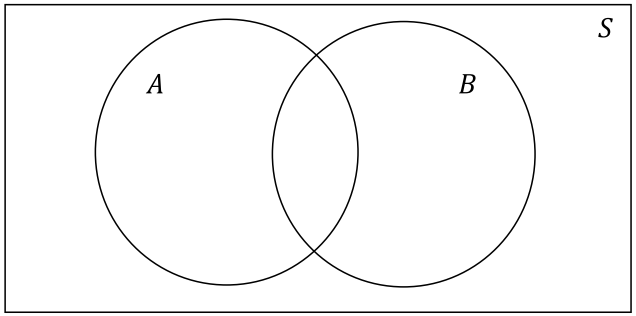
$P(\sim A)$



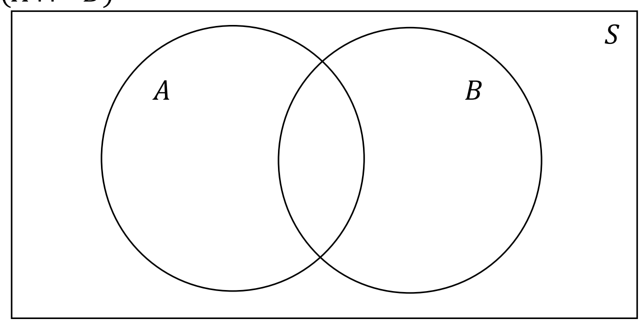
$P(A \cap B)$



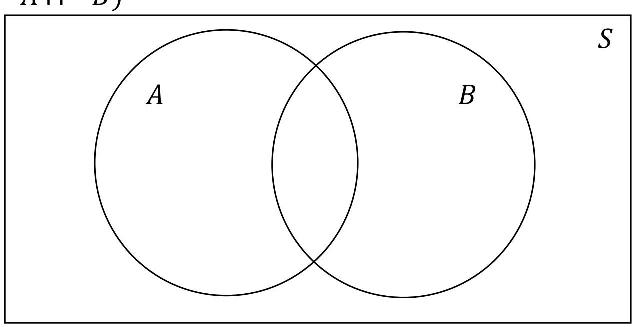
$P(A \cup B)$



$P(A \cap \sim B)$



$P(\sim A \cap \sim B)$



The ***Addition Rule*** is used to find "OR" probabilities.

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Let's Practice!

1. Suppose it is known that 80% of high school students are Harry Potter fans, 30% are Twilight fans, and 20% are fans of both.

   a. What is the probability that a randomly selected student does not like Harry Potter?

   b. What is the probability that a randomly selected student likes Harry Potter but does not like Twilight?

   c. What is the probability that a randomly selected student likes Harry Potter or Twilight?

   d. What is the probability that a randomly selected student does not like Harry Potter or Twilight?

   e. Draw a tree of the possible outcomes, determine the sample space, and find the associated probabilities.

Section 11 – Topic 4
Probability and the Addition Rule – Part 2

Try It!

1. Ron and Harry are first-year students at Hogwarts and are going to be placed into one of four houses: Gryffindor, Ravenclaw, Hufflepuff, and Slytherin. Each house is equally likely.

   a. Determine the sample space.

   b. What is the probability that Ron gets placed in Gryffindor?

   c. What is the probability that Harry does not get placed in Gryffindor?

   d. What is the probability that at least one of them gets placed in Gryffindor?

2. Complete the two-way frequency table below about students at the last school dance:

Gender	Grade 9	Grade 10	Grade 11	Grade 12	Total
Boys	6	15	35	40	
Girls	11	17	39	41	
Total					

Suppose a student is randomly selected:

   a. Determine $P(Girl)$.

   b. Determine $P(11th\ Grader)$.

   c. *Determine* $P(11th\ Grader\ and\ girl)$.

   d. Determine $P(11th\ or\ girl)$.

BEAT THE TEST!

1. Consider a standard deck of 52 playing cards. There are 52 cards in a standard deck of cards – half are red (hearts and diamonds), half are black (spades and clubs), and of the 52 cards, 12 are "face" cards (jacks, queens and kings). Aces are considered to have a value of one.

A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦

Part A: Suppose a card is randomly selected. What is the probability of drawing a face card?

Part B: Suppose a card is randomly selected. What is the probability of drawing a red face card?

Part C: Suppose a card is randomly selected. What is the probability that the card is red or a face card?

2. An ice cream stand has vanilla, chocolate, strawberry, and butter pecan flavors. They also have hot fudge, gummy bears, sprinkles, crushed pecans, and marshmallows as toppings. The stand is running a special: one flavor and one topping for 99 cents. Assume that all flavors and toppings are equally popular.

Part A: How many combinations of ice cream and topping are available?

Part B: What is the probability that a customer orders chocolate ice cream?

Part C: What is the probability that a customer's order is for chocolate ice cream with sprinkles?

Part D: What is the probability that a customer's order has chocolate ice cream or sprinkles?

Section 11 – Topic 5
Probability and Independence

Two events are ***independent*** if the outcome of the first event does not affect the outcome of the second.

Consider flipping a fair coin.

On the first toss, what is the probability of heads?

On the second toss, what is the probability of heads?

If the first flip results in heads, what is the probability of heads on the second flip?

Are the coin flips independent? Why or why not?

Consider dealing a hand from a deck of cards.

What is the probability an ace is drawn on the first card?

What is the probability an ace is drawn on the second card?

Is getting an ace on the first and second cards independent? Why or why not?

Identify each example as independent or dependent.

Flipping a coin, and then flipping it again	○ Dependent ○ Independent
Choosing a black 7 from a deck of cards and not returning it, then choosing another black 7	○ Dependent ○ Independent
Pulling a blue M&M from a package of candy and then pulling a brown M&M from the same package	○ Dependent ○ Independent
A couple's first child having red hair and their second child having red hair	○ Dependent ○ Independent

➢ If two events are independent, then $P(A \cap B) = P(A) * P(B)$.

Let's Practice!

1. Suppose it is known that 80% of high school students are Harry Potter fans, 30% are Twilight fans, and 20% are fans of both.

   a. Are these events independent?

   b. Now, suppose it is known that 80% of high school students are *Harry Potter* fans, 30% are *Twilight* fans, and the events are independent.

      What is the probability of randomly selecting a student who likes *Harry Potter* and *Twilight?*

Try It!

2. Bailey is working at her local clothing store. Based on past sales, the probability that the next item purchased will be a dress is 0.20. Additionally, the probability that the next item purchased will be something red is 0.30.

   a. Assuming there are no red dresses, are buying a red item and buying a dress independent?

   b. What is the probability that the next item purchased is a dress or something red?

BEAT THE TEST!

1. Mike makes 85% of the free throws he attempts in basketball. Suppose Mike gets two free throws and his free throws are independent.

   Part A: Draw a tree diagram of the possible outcomes, determine the sample space, and find the associated probabilities.

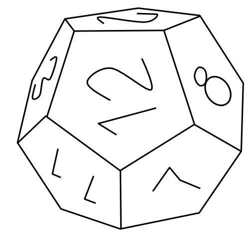
   Part B: What is the probability Mike makes both free throws?

   Part C: What is the probability that Mike makes at least 1 free throw?

   Part D: Suppose Mike makes the first free throw, what is the probability that he makes the second free throw?

Section 11 – Topic 6
Conditional Probability

Recall the dodecahedral die, which is a 12-faced die where each face is numbered from 1 to 12.



Event A = rolling an even number
Event B = rolling a number greater than eight

$P(A) = {}^{6}/_{12} = 0.50$

$P(B) = {}^{4}/_{12} = 0.33$

What is the probability of rolling an even number that is greater than eight?

If you roll a number greater than eight, what is the probability that it is an even number?

If you roll an even number, what is the probability that is a number greater than eight?

Rather than comparing a value to the whole population, or the sample size, ***conditional probabilities*** look at a specific subgroup.

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Recall that events A and B are ***independent events*** if and only if the outcome of one does not affect the outcome of the other.

- Two events are independent if and only if:

$$P(A|B) = P(A)$$
or
$$P(B|A) = P(B)$$

- This means that when two events are independent, the occurrence of one has no effect on the probability of the other event occurring.

Are rolling an even number and rolling a number greater than eight independent? Justify your answer.

Let's Practice!

1. Suppose it is known that 80% of high school students are *Harry Potter* fans, 30% are *Twilight* fans, and 20% are fans of both.

   a. Given that a student is a *Harry Potter* fan, what is the probability that the student is a *Twilight* fan?

   b. What is the probability that a student is a *Harry Potter* fan if the student is a *Twilight* fan?

   c. Are *Harry Potter* fandom and *Twilight* fandom independent? Justify your reasoning.

BEAT THE TEST!

1. A survey of students at Adams School for the Arts revealed that 62% prefer classical music when studying over no music at all. In addition, 37% indicated that they are left-handed. Of those surveyed, 14% said they are left handed and do not prefer classical music.

   Part A: Complete the table below:

	Left-handed (L)	**Not Left-handed ($\sim L$)**	**Total**
Prefer Classical Music (C)			
Do Not Prefer Classical Music ($\sim C$)			
Total			

   Part B: Given that a randomly selected student is not left-handed, what is the probability that he/she prefers classical music?

   Part C: Find $P(\sim C|L)$.

   Part D: Describe what $P(L|\sim C)$ means.

   Part E: If a person is left-handed, are they more likely to prefer classical music than someone who is not left-handed?

   Part F: Are left-handedness and preferring classical music independent? Justify your answer.

Two-Way Frequency Tables – Part 1

Determine the following conditional probabilities using the table given below.

	Males (M)	Females (F)	Total
Has an afterschool job (J)	182	143	325
Does not have an afterschool job $(\sim J)$	98	77	175
Total	280	220	500

A randomly selected student is male. What is the probability that he has an afterschool job?

What is the probability that a student has an afterschool job, given that she is female?

Do you think that male students are more likely to have an afterschool job? Justify your answer.

We can also write these conditional probabilities using the notation for intersections that we learned earlier.

The probability that a randomly selected student has an afterschool job given they are a male can be rewritten as $P(J|M)$.

The formula for this is $P(J|M) = \frac{P(J \text{ and } M)}{P(M)} = \frac{P(J \cap M)}{P(M)}$

Use symbolic notation to find the probability that a randomly selected student who does not have an afterschool job is a female.

Let's Practice!

1. Shown below is a contingency table summarizing the casualty figures from the RMS Titanic disaster by passenger class, as recorded by the United States Senate Inquiry:

	Perished	Survived	Total
1st class	130	199	329
2nd class	166	119	285
3rd class	536	174	710
Crew	685	214	899
Total	1517	706	2223

   a. What percentage of passengers had 1st class tickets?

   b. What percentage of passengers had 1st class tickets and survived?

   c. What percentage of 1st class ticket passengers survived?

   d. Does there seem to be a relationship between passenger class and survival? Justify your answer.

Section 11 – Topic 8
Two-Way Frequency Tables – Part 2

Try It!

1. The local Humane Society is making a new calendar for this year's fundraiser. They surveyed 200 of their contributors and found that 70% would purchase a calendar, 40% consider themselves a "dog person", and 12% would not purchase a calendar and do not consider themselves a "dog person".

   a. Complete the table below with the counts for each cell.

	Dog Person	Not a Dog Person	Total
Would purchase			
Would not purchase			
Total			

   b. Given that a randomly selected contributor is a "dog person," what is the probability that this contributor would purchase a calendar?

   c. What is the probability that a contributor who would not purchase the calendar is a "dog person?"

   d. Are these events independent? Justify your answer.

BEAT THE TEST!

1. Artificial turf on athletic fields was first introduced in the 1960s. Its safety has been controversial since then. One issue that has been investigated is whether injuries of football players tend to be more serious on artificial turf than on grass. A study followed 24 NCAA Division 1A college football teams over three seasons.

   In total, there were 1,050 injuries that occurred on field turf. Of the field turf injuries, $83.\bar{3}\%$ were minor and 10.857% were substantial.

   77.972% of grass injuries were minor.

   4.26% of injuries occurred on grass and were severe.

   Complete the table below.

	Field Turf	Grass	Total
Minor			1,813
Substantial			
Severe			
Total	1,050		

2. A clinic runs a test to determine whether or not patients have a particular disease. The test is notorious for its inaccuracy. The two-way frequency table below summarizes the numbers of patients in the past year that received each result.

	Positive result	Negative result	Total
Has disease	150	6	156
Does not have disease	18	840	858
Total	168	846	1014

Part A: A patient from this group received a positive test result. What is the probability that he or she has the disease?

Part B: A patient from this group has the disease. What is the probability that he or she received a positive result on the test?

Part C: A "false positive" is when a patient receives a positive result on the test, but does not actually have the disease. What is the probability of a false positive for this sample space?

Section 11 – Topic 9
Empirical and Theoretical Probability

There are many approaches to finding probabilities. Suppose we want to determine the probability of getting heads when flipping a coin.

One way to find $P(heads)$ is

$$P(Event) = \frac{\#\ of\ outcomes\ in\ the\ event}{\#\ of\ outcomes\ in\ the\ sample\ space} =$$

$$P(heads) = \frac{heads}{heads,\ \ tails} = \frac{1}{2} = 0.5$$

This approach is called ***Theoretical Probability***.

- The theoretical probability of an event is the number of ways the event can occur, divided by the total number of outcomes.

- It comes from a sample space of equally likely outcomes.
  - Flipping a coin, rolling a fair die, etc.

There are other ways to estimate probability. The ***Empirical Probability*** of an event is determined by the proportion of times the event will occur in a long series of independent trials.

- Toss a coin 10 times and observe 8 heads $\rightarrow$ $P(Heads) = \frac{8}{10}$

- Toss a coin $10{,}000$ times and observe $5{,}020$ heads $\rightarrow$ $P(Heads) = \frac{5020}{10000} = 0.50$

According to the ***Law of Large Numbers***, as the sample size increases, the empirical probability will approach the theoretical probability.

Let's Practice!

1. Jamie and Marcus are rolling two dice 30 times. The sums of their rolls are shown below:

$9, 10, 8, 8, 4, 5, 6, 4, 6, 5, 8, 7, 6, 9, 2, 4, 6, 6, 7, 7, 5, 8, 7, 6, 7, 4, 10, 8, 8, 3$

   a. What is the empirical probability of rolling a sum of six?

   b. What is the theoretical probability of rolling a sum of six?

   c. How do the empirical and theoretical probabilities compare?

Try It!

2. Karyn is spinning a prize wheel divided into three equal sections: the first is a free t-shirt, the second is a pencil, and the third is spin again. Karyn spins the wheel 15 times and gets the following results:

Spin Again	Spin Again	Pencil
T-Shirt	T-Shirt	Pencil
Spin Again	Spin Again	Pencil
T-Shirt	T-Shirt	Spin Again
T-Shirt	Pencil	Spin Again

   a. What is the theoretical probability of spinning the wheel and winning a pencil?

   b. What is the empirical probability of winning a pencil?

BEAT THE TEST!

1. You assign three different groups to flip a coin repeatedly. Group A is going to flip the coin 50 times; Group B is going to flip the coin 200 times; Group C is going to flip the coin 400 times. Which group is likely to have results that are farthest from the theoretical probability?

   - o Group A
   - o Group B
   - o Group C

2. The table below represents the distribution of your class rolling a number cube 15 times. Does it represent the theoretical probability of rolling a number cube? Why or why not?

Outcome	1	2	3	4	5	6
Frequency	1	2	6	1	2	3

Test Yourself! Practice Tool

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!

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