Section 8: Expressions and Equations with Radicals and Rational Exponents

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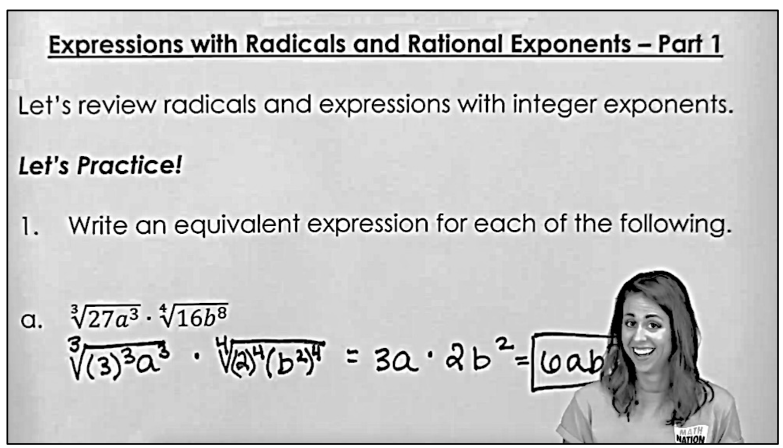


The following Mathematics Florida Standards will be covered in this section:
A-CED.1.1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational, absolute, and exponential functions.
A-CED.1.4 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law, $V = IR$, to highlight resistance, R.*
A-REI.1.2 - Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
F-BF.2.3 - Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-IF.2.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F-IF.2.5 - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
F-IF.3.7b - Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F-IF.3.9 - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
N-RN.1.1 - Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$, to hold, so $(5^{1/3})^3$ must equal 5.*
N-RN.1.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Section 8: Expressions and Equations with Radicals and Rational Exponents
Section 8 – Topic 1
Expressions with Radicals and Rational Exponents – Part 1

Let's review radicals and expressions with integer exponents.

Let's Practice!

1. Write an equivalent expression for each of the following expressions.

a. $\sqrt[3]{27a^3} \cdot \sqrt[4]{16b^8}$

b. $\sqrt[4]{81x^2y^5}$

c. $\sqrt{\dfrac{4x^4}{(256y^8)^{-2}}}$

d. $\left(\dfrac{x^{-5}y^{-3}}{z^3}\right)^{-5}$

e. $\dfrac{4y^{3x-3}}{2y^{2x+4}}$

Try It!

2. Write an equivalent expression for each of the following.

   a. $\sqrt[5]{-32x^{10}y^5} \cdot \sqrt[3]{8x^{21}y^6}$

b. $\sqrt[3]{\dfrac{27x^9}{(216z^6)^{-3}}}$

c. $\dfrac{6^{-1}a^2b^{-3}}{3^{-2}a^{-5}b^2}$

d. $\dfrac{a^{3b+2}\cdot a^{2b}}{a^{-2b-5}}$

Let's review rational exponents.

Let's explore why $\sqrt[3]{5} = 5^{\frac{1}{3}}$.

$\left(\sqrt[3]{5}\right)^3 = \sqrt[3]{5}\cdot\sqrt[3]{5}\cdot\sqrt[3]{5} = \sqrt[3]{125} =$ _________

$\left(5^{\frac{1}{3}}\right)^3 = 5^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}\cdot 5^{\frac{1}{3}} = 5^{\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)} =$ _________

Hence, $\sqrt[3]{5} =$ ________.

Let's Practice!

3. Explain why $\sqrt[3]{6^2} = 6^{\frac{2}{3}}$.

Definition of Rational Exponents: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Section 8 – Topic 2
Expressions with Radicals and Rational Exponents – Part 2

Let's Practice!

1. Rewrite the following expressions using rational exponents.

   $(6d)^{\frac{1}{4}} =$

2. Rewrite the following expressions using rational exponents.

   a. $\sqrt[3]{(x+2)^4}$

   b. $\sqrt[6]{y} \cdot \sqrt[3]{y} \cdot \sqrt[5]{y^2}$

   c. $\sqrt[3]{\dfrac{8x^7}{(27y^5)^{-1}}}$

   d. $\sqrt{\sqrt[3]{(y+1)^4}}$

Try It!

3. Rewrite the following expression using radicals.

   $(10y)^{\frac{2}{3}}$

4. Rewrite the following expressions using rational exponents.

   a. $\sqrt[4]{(2x-3)^3}$

   b. $\sqrt[4]{\sqrt{(x-1)^5}}$

   c. $\sqrt{\dfrac{4x^{11}}{(256y^8)^{-2}}}$

BEAT THE TEST!

1. Given that y is a negative integer, which of the following statements is always true?

   Ⓐ $5y^{-2} < 4y^{-1}$
   Ⓑ $5y^{-1} < 4y^{-1}$
   Ⓒ $\frac{y}{4} > -5y^{-1}$
   Ⓓ $4y^{-1} > (5y)^{-1}$

2. Consider the following equation with radicals and rational exponents.

$$11^{\frac{19}{4}} \cdot \sqrt[a]{11^b} = 11^{\frac{9}{4}} \cdot \sqrt{11^3}$$

   What is the quotient of b and a?

3. What is the value of $\sqrt{5x^{\frac{1}{2}} + \sqrt{x^0} + x^{\frac{1}{4}}}$ when $x = 81$?

Section 8 – Topic 3
Solving Equations with Radicals and Rational Exponents – Part 1

We can solve equations with radicals and rational exponents by transforming them into equations that do not have radicals or rational exponents.

In each of the following expressions, what operation would we perform to get the variable raised to the first power?

$\sqrt{x} = 3$

$x^2 = 5$

$x^{\frac{1}{5}} = 2$

Let's Practice!

1. Solve the following equations.

   a. $6\sqrt{x-4} = 48$

   b. $(3x-5)^{\frac{1}{4}} = 20$

Try It!

2. Solve the following equations.

   a. $5\sqrt{x+13} - 4 = 46$

   b. $(2x-3)^{\frac{1}{2}} = 17$

Sometimes we may find extraneous solutions.

Consider the following equation.

$$\sqrt{4-x} = x-2$$

Transform the equation so that it does not have radicals and find the solution(s) to the new equation.

Are both solutions to the transformed equation solutions to the original equation? Justify your answer.

Let's Practice!

3. Solve the following equation. Check for extraneous solutions.

   $x - \sqrt{x - 2} = 4$

Try It!

4. Solve the following equation. Check for extraneous solutions.

   $-\sqrt{1 - x} + x + 5 = 0$

Section 8 – Topic 4
Solving Equations with Radicals and Rational Exponents – Part 2

A software programmer needs to write an algorithm for computing distance, d, using a formula that computes time, t. The formula for time, t, is shown below.

$$t = \sqrt{2d/a}$$

What is the formula for d that the programmer will use to write the algorithm?

The programmer also needs to write a formula for acceleration, a. What is the formula for a that the programmer will use to write the new algorithm?

Let's Practice!

1. The average consumption of tomatoes by Italians (in pounds per person) between 1980 and 2000 can be modeled by equation $t = \sqrt{22y + 275}$, where t is tomatoes in pounds per person and y is the number of years since 1980.

   a. What equation can be used to calculate the year when a specific number of pounds of tomatoes were consumed per person?

   b. In what year were 20 pounds of tomatoes consumed per person?

2. Isaac Newton discovered how to calculate the minimum speed needed for an object to "break free" from the gravitational attraction of a massive body. This is called the escape velocity and it is measured with $V = \sqrt{\frac{2GM}{R}}$, where V is the escape velocity, G is the gravitational constant, M is the mass of the body, and R is the radius of the body.

   a. What is the formula *for* R, the radius of the body?

   b. What is the formula for M, the mass of the body?

Try It!

3. The swing of a pendulum measured in seconds, S, can be modeled by equation $S = 2\pi\sqrt{\frac{L}{32}}$, where L is the length of the pendulum (in feet).

   Find the approximate length of the pendulum that makes one swing in 3.5 seconds. Round your answer to the nearest hundredth of a foot.

BEAT THE TEST!

1. The formula $KE = \frac{1}{2}mv^2$ represents the kinetic energy, KE, of an object, where m is the mass of the object, and v is the velocity of the object. Which of the following gives v as a function of KE and m?

   Ⓐ $v = \left(\frac{2KE}{m}\right)^2$

   Ⓑ $v = \sqrt{\frac{2KE}{m}}$

   Ⓒ $v = \left(\frac{2m}{KE}\right)^2$

   Ⓓ $v = \sqrt{\frac{2m}{KE}}$

2. Solve the equation below for x.

$$\frac{(x-11)^{\frac{1}{2}}}{7} = 1$$

$x =$ ☐

3. Which value for the constant c makes $x = -\frac{10}{3}$ an extraneous solution in the following equation?

$$\sqrt{-3x-1} = xc + 2$$

   Ⓐ $-\frac{2}{3}$

   Ⓑ $-\frac{10}{3}$

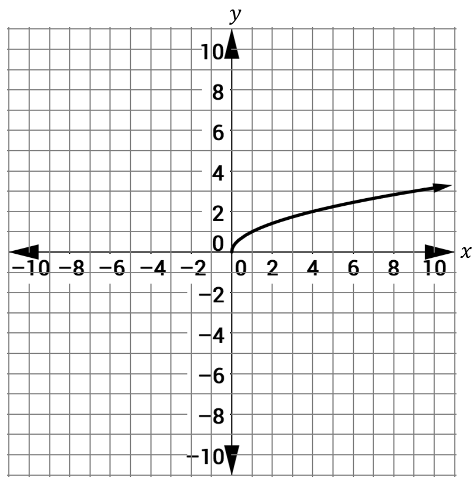
   Ⓒ $\frac{3}{10}$

   Ⓓ $\frac{3}{2}$

Section 8 – Topic 5
Graphing Square Root and Cube Root Functions – Part 1

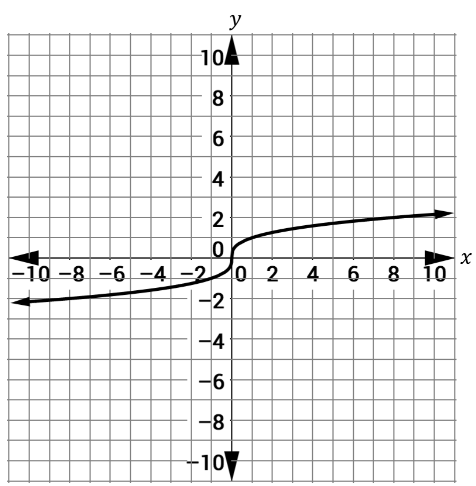
Consider the square root functions below.

$$f(x) = \sqrt{x}$$



Consider the cube root functions below.

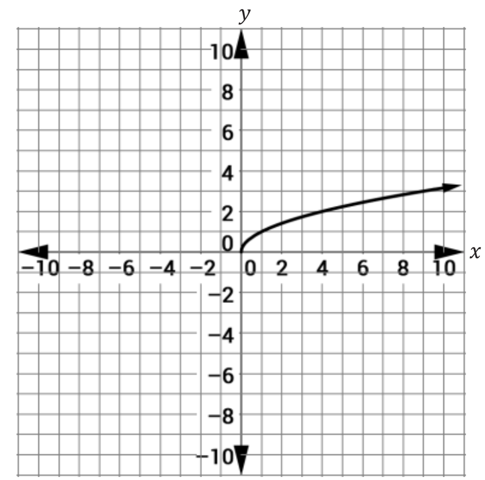
$$f(x) = \sqrt[3]{x}$$



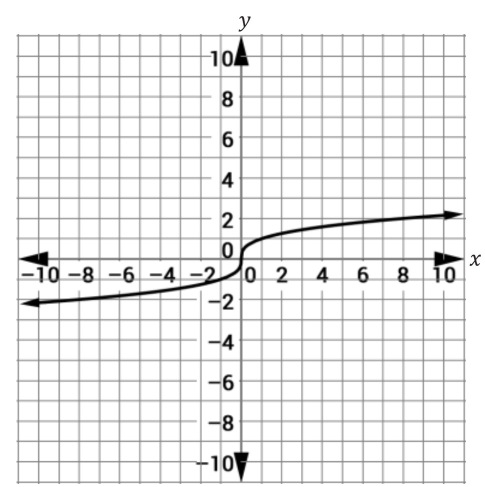
Let's Practice!

1. Use your knowledge of transformations to sketch the graphs of the following functions on the same coordinate plane.

   a. $f(x) = \sqrt{x-2} + 3$



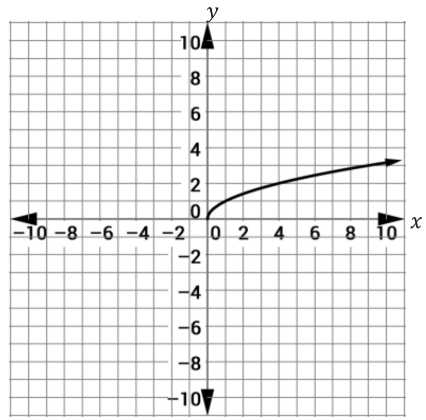
   b. $f(x) = \sqrt[3]{x} - 2$



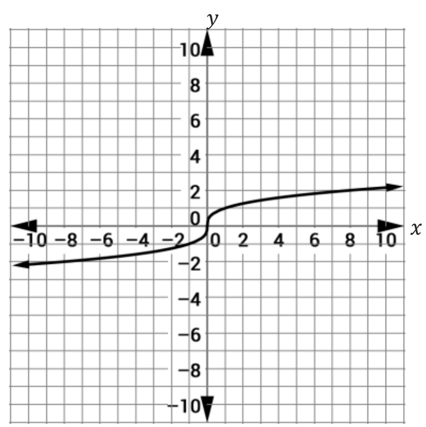
Try It!

2. Use your knowledge of transformations to sketch the graphs of the following functions on the same coordinate plane.

   a. $g(x) = \sqrt{x+5} - 4$



   b. $h(x) = \sqrt[3]{x-1} + 2$



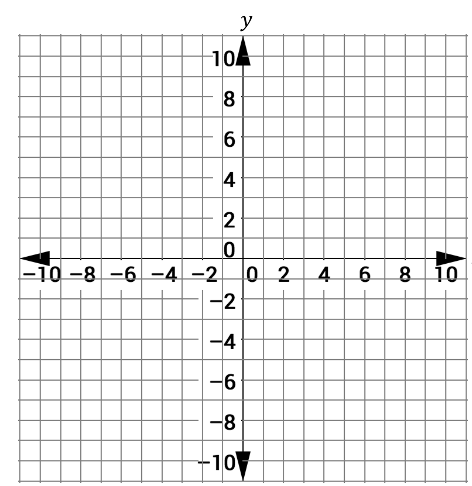
Section 8 – Topic 6
Graphing Square Root and Cube Root Functions – Part 2

The Rotor is an amusement park ride that was designed in the late 1940s. Riders stand against a circular wall. When the ride reaches a certain speed, the floor drops. The centrifugal force keeps the riders pinned to the wall. The function that models the speed necessary to keep someone pinned to the wall is given by $s(r) = \sqrt{4.95r}$, where r is the radius, in meters, of the Rotor.

What is a feasible domain for the function?

What is a feasible range for the function?

Sketch the graph that models the speed with respect to the radius.



What is the necessary speed to keep a person pinned to the wall on the Rotor that has a 9-meter radius?

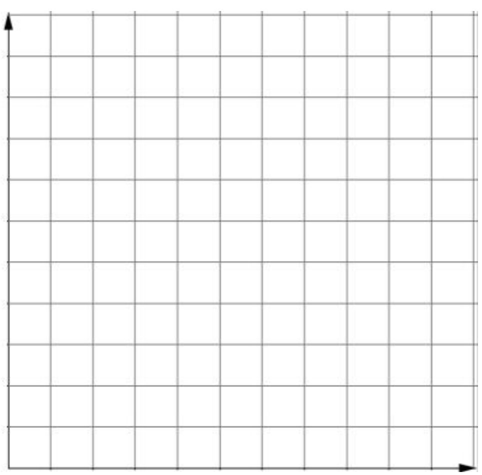
Try It!

1. The shoulder height, in centimeters, of a male African elephant can be modeled by the function $h(t) = 62.5\sqrt[3]{t} + 75.8$, where t is the age of the elephant in years.

   a. What is a feasible domain for the function?

   b. What is a feasible range for the function?

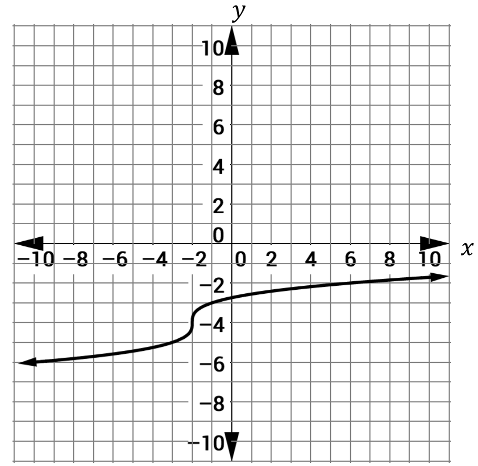
   c. Sketch the graph of an elephant's height with respect to age.



   d. Find the difference of the shoulder height of a ten-year-old elephant and a five-year-old elephant.

BEAT THE TEST!

1. Which of the following functions matches the graph below?



   Ⓐ $y = \sqrt[3]{x+2} + 4$
   Ⓑ $y = \sqrt[3]{x+2} - 4$
   Ⓒ $f(x) = \sqrt[3]{x-2} + 4$
   Ⓓ $f(x) = \sqrt[3]{x-2} - 4$

2. Consider the following function.

$$y = -\sqrt{x-2} + 3$$

   Which of the following statements are true? Select all that apply.

   ☐ The graph is a cube root function.
   ☐ The domain of the function is $[2, \infty)$.
   ☐ The range of the function is $[3, \infty)$.
   ☐ The graph has an x –intercept at $(11, 0)$.
   ☐ The graph has a y –intercept at $(0, 3)$.
   ☐ The graph is decreasing.